Solutions des exercices sur les intégrales

(1) a) $$\int \frac{\cos x}{\sin^4 x}\,dx = \int \cos x \cdot \sin^{-4} x \cdot dx$$

$$= \frac{\sin^{-3} x}{-3} + C = \frac{1}{-3\sin^3 x} + C$$

b) $$\int \frac{2}{\sqrt{9-4x^2}}\,dx = \int \frac{2}{3\sqrt{1-\frac{4x^2}{9}}}\,dx \quad (*)$$

$$= \frac{2}{3} \cdot \arcsin\left(\frac{2x}{3}\right) \cdot \frac{3}{2} + C$$

$$= \arcsin\left(\frac{2x}{3}\right) + C$$

ou par changement de variable : $u = \frac{2x}{3} \rightarrow du = \frac{2}{3}\,dx$

$$(*) = \int \frac{2}{3\sqrt{1-u^2}} \cdot \frac{3}{2}\,du = \arcsin u + C = \arcsin\left(\frac{2x}{3}\right) + C$$

c) $$\int 6x \cdot \sqrt{3x+4}\,dx$$

$u = x \rightarrow u' = 1$

$v' = (3x+4)^{1/2} \rightarrow v = \frac{(3x+4)^{3/2}}{\frac{3}{2}} \cdot \frac{1}{3}$

$v = \frac{2}{9}(3x+4)^{3/2}$

$$= 6 \cdot \left[x \cdot \frac{2}{9}(3x+4)^{\frac{3}{2}} - \frac{2}{9}\int (3x+4)^{3/2}\,dx\right]$$

$$= \frac{4}{3}x(3x+4)^{\frac{3}{2}} - \frac{4}{3} \cdot \frac{(3x+4)^{5/2}}{\frac{5}{2}} \cdot \frac{1}{3} + C$$

$$= \frac{4}{3}x \cdot (3x+4)^{\frac{3}{2}} - \frac{8}{45}(3x+4)^{5/2} + C$$

ou par changement de variable : $u = 3x+4 \rightarrow du = 3 \cdot dx$

$$6\int \frac{u-4}{3}\sqrt{u}\,\frac{du}{3} = \frac{2}{3}\int \left(u^{\frac{3}{2}} - 4u^{\frac{1}{2}}\right)du$$

$$= \frac{2}{3} \cdot \left[\frac{(3x+4)^{5/2}}{\frac{5}{2}} - 4 \cdot \frac{(3x+4)^{3/2}}{\frac{3}{2}}\right] + C = \frac{4}{15}(3x+4)^{\frac{5}{2}} - \frac{16}{9}(3x+4)^{\frac{3}{2}} + C$$